NEIP-95-003 March 1995

Evidence for the Validity of the Berry-Robnik Surmise in a Periodically Pulsed Spin System [1]

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[1]Classification Numbers : 0545 / 0365

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Abstract

We study the statistical properties of the spectrum of a quantum dynamical system whose classical counterpart has a mixed phase space structure consisting of two regular regions separated by a chaotical one. We make use of a simple symmetry of the system to separate the eigenstates of the time-evolution operator into two classes in agreement with the Percival classification scheme [2]. We then use a method firstly developed by Bohigas et. al. [3] to evaluate the fractional measure of states belonging to the regular class, and finally present the level spacings statistics for each class. The level spacings distribution of states belonging to the irregular part of the spectra as well as that of the complete set of levels corroborate the Berry-Robnik surmise [6]. We further present a statistical study of the regular levels. The presence of intermediate states - states which belong to neither class as long as $\hbar$ is finite, phase spatially mixed among the set of regular ones, together with the small fractional measure of regular states strongly affects the corresponding level spacings statistics, resulting in a non negligible deviation from the expected Poisson distribution. We see however the remarkable agreement of the irregular level spacings statistics as a direct confirmation of the Berry-Robnik surmised.

1 Introduction

For more than a decade, the study of quantum mechanical systems whose classical counterpart exhibits chaos has attracted much interest. One motivation for this study is the paradoxical fact that while the correspondence principle, as we understand it, should imply a quantum manifestation of classical chaos, the Schrödinger equation is linear. As a consequence, the time-evolution operator is unitary, and this suppresses any exponential divergence in the time evolution of quantum states. As a spectacular manifestation of this fact, time-reversal invariant models show no lost of memory : reversing the time at a certain moment T brings us back to the initial situation after another time interval T, while this would require infinite precision in a classical chaotic system. Thus a basic manifestation of classical chaos seems to have no place in quantum mechanics.
On the other hand, the destruction of an integral of motion, of a quantum number, has striking effects on the statistical properties of quantum spectras. It is today taken as granted that in a classically integrable system, the levels are uncorrelated, and so have a poissonian level spacings distribution [4] (a remarkable exception being the one-dimensional harmonic oscillator) and that in classically fully chaotic models, the level spacings distribution has a dramatically different shape : it obeys predictions of random matrix theory, i.e. it exhibits level repulsion [1]. The situation in mixed systems, where regular and chaotic regions coexist in the classical phase space, is more intricated. In an old paper Percival [2] classified the eigenfunctions of the Schrödinger equation into two classes belonging to either the regular regions, where the invariant tori are not destroyed, or the chaotic one. This classification was based mostly on the correspondence principle and has been numerically confirmed a few years ago by Bohigas et. al. [3]. While the eigenfunctions that are mostly confined on classically regular regions - we will call them the regular eigenfunctions - tend to concentrate on invariant tori, the irregular ones tend to spread uniformly over the chaotic region as $\hbar \to 0$, as has been rigorously demonstrated by Shnirelman [12]. This picture is assumed to reflect reality in the semiclassical limit $\hbar = 0$. Following this classification, Berry and Robnik postulated that the part of the spectrum that corresponds to regular eigenfunctions, has a poissonian level spacings distribution in opposition to the one corresponding to the irregular eigenstates which exhibits level repulsion [6]. This surmise led them to an expression for the level spac-

ings distribution for mixed systems that has been observed convincingly only recently [7] for the case of the kicked rotator on a torus. As pointed out by Prosen & Robnik and Li & Robnik [8] [13], reasons for this difficulty of observation could be that we are not deep enough in the semi-classical regime. As long as $\hbar$ is finite, a certain number of wave-functions belong neither to the regular nor to the irregular set of eigenfunctions. We may think of states making use of the Heisenberg uncertainty to overlap the frontier between the regular and irregular regions of the classical phase space, or states located on the regular region which, due to the finiteness of the Planck constant, do not yet belong to the set of regular states. Consequently, the Berry-Robnik regime should be observable only in the far semiclassical limit. We will come back to this point later.

In this paper we present a spin model allowing a precise study of a mixed regime. The reasons for this are first that, in a special regime, an approximate simple symmetry of the phase space structure, namely $S_z \to -S_z$, allows the separation of regular states from the irregular ones, and secondly that the frontier between the regular and the chaotic zones is rather sharp, thus minimizing the number of intermediate eigenstates. This enables us to compute the level spacings statistics independently for the regular and irregular states. We interpret the fact that these statistics obey quite well the poissonian distribution and the GOE respectively as a direct confirmation of the Berry-Robnik surmise.

We study the quantum system defined by the following Hamiltonian [4]:

$$\mathbf{H}_{qm} := \frac{m}{2}((1-z^2)\mathbf{S}_z^2 - z^2\mathbf{S}_x^2) + \kappa\mathbf{S}_z\Delta_T \tag{1}$$

and the corresponding unitary time evolution operator :

$$\mathbf{U}_T := \exp(-\frac{i}{\hbar}\kappa\mathbf{S}_z)\exp(-\frac{i}{\hbar}\frac{m}{2}((1-z^2)\mathbf{S}_z^2 - z^2\mathbf{S}_x^2)T) \tag{2}$$

where $\vec{\mathbf{S}} = (\mathbf{S}_x, \mathbf{S}_y, \mathbf{S}_z) = \hbar(\mathbf{s}_x, \mathbf{s}_y, \mathbf{s}_z) = \hbar\vec{\mathbf{s}}$ are spin operators satisfying the usual commutation rules (ϵ^{ijk} is the total antisymmetric tensor of 3^{rd} order):

$$[\mathbf{S}_i, \mathbf{S}_j] = i\hbar\epsilon^{ijk}\mathbf{S}_k \tag{3}$$

[4]We use bold characters in the quantum case in contrast to normal ones which refer to the classical and semiclassical cases : $\vec{\mathbf{S}}$ refers to the quantum spin operator while $\vec{S}$ is either a classical or a semiclassical spin.

$0 \leq \kappa \leq 2\pi$, $\Delta_T := \sum_{n=-\infty}^{+\infty} \delta(t - nT)$, $[m]$ = energy^{-1} time^{-2} and $0 \leq z \leq 1$. Models of this kind have been extensively studied [11]. They represent a spin which evolves under the influence of a classically integrable Hamiltonian $\mathbf{H}_{qm}^0 = \frac{m}{2}((1-z^2)\mathbf{S}_z^2 - z^2\mathbf{S}_x^2)$ during a time T after which the spin undergoes a rotation of angle κ around the z-axis. The regime we consider is defined by $\kappa = 1.1$, $T = \frac{19}{mS}$ and $\mathrm{z}^2 = \frac{1}{2}$. Classically there are two regular zones around the north and south poles surrounding a chaotic region which is fairly well symmetric under S_z reflection (Fig.1). In the semiclassical limit which corresponds to $\hbar s = S$ = constant, $\hbar = s^{-1} \to 0$, states which are located on the chaotic region tend to cover it homogeneously according to Shnirelman's theorem [12]. Since this region is symmetric under S_z reflection, the expectation value $< \Psi_{chaos}|\mathbf{s}_z|\Psi_{chaos} >$ of such a state tends to disappear as we approach the semi-classical limit. For small but finite $\hbar$, the distribution of $< \Psi_k|\mathbf{s}_z|\Psi_k >$, where $|\Psi_k >$ is an eigenstate of the operator $\mathbf{U}_T$ defined in (2), will then present a sharp peak around zero corresponding to the irregular states surrounded by two smaller bumps corresponding to regular states (Fig.2). This allows us to separate easily the regular states from the irregular ones, the validity of this selection being confirmed by a numerical semiclassical argument presented in section 3 as well as an extensive study of the Husimi distributions of the selected states [15].

The paper is organized as follows : Section 2 is devoted to a short presentation of the classical model. In section 3 we derive some useful semiclassical quantities such as the density of states and the expression for the action. This will allow us to estimate the number of regular states, and give a check of our selection criterion. In section 4 we present the quantum mechanical model as well as our numerical results for a spin magnitude s=500. All of them were obtained using direct diagonalization techniques. Conclusions and further remarks are given in section 5.

2 Classical model

The unperturbed classical Hamiltonian

$$H_{cl}^0 := \frac{m}{2}((1-z^2)S_z^2 - z^2 S_x^2) \tag{4}$$

has two degrees of freedom and is an integral of motion. The trajectories are confined to the intersections of the sphere $|\vec{S}| = S$ with the cones of constant

energy $E = \frac{m}{2}((1-z^2)S_z^2 - z^2 S_x^2)$. The perturbation :

$$H_{cl}^1 := \kappa S_z \Delta_T \tag{5}$$

corresponds to a rotation of angle κ around the z-axis performed at time intervals T. Its addition leads to the destruction of the energy surfaces, and allows more and more trajectories to wander chaotically on the sphere of constant spin magnitude as κ and T grow. Expanding H_{cl}^0 up to the first order in $\delta S_z := S - S_z$ near the poles $S_z = \pm S$,we get a one-dimensional harmonic oscillator of period $T = 2\sqrt{2}\frac{\pi}{mS}$. In particular we have $\delta \dot{S}_z = O(\delta S_z^2)$: in this approximation δS_z is an integral of motion and is furthermore conserved by the perturbation H_{cl}^1 too. It is thus conceivable that the invariant torii near the poles will offer more resistance to the perturbation than those located away from them. We use this property to find a regime in which there are two regular islands around the poles approximately related by the operation $S_z \to -S_z$ and separated by a chaotic region. This we achieved by setting $\kappa = 1.1, T = \frac{19}{mS}, z^2 = 0.5$ (Fig.1). The regular islands occupy in a good approximation the region $0.22S^2 \leq E \leq E_{max} = 0.25S^2$.

3 Semiclassical approach

We compute the Green function for a trajectory of positive energy and the density of states for the unperturbed case $T = \frac{19}{mS}, z^2 = 0.5$. We follow the lines drawn in [9]. We first write the unperturbed Hamiltonian in canonical variables (S_z, ϕ) for the chosen regime :

$$H_0 = \frac{m}{4}(S_z^2(1+\cos^2(\phi)) - \vec{S}^2\cos^2(\phi)) \tag{6}$$

The action integral for a trajectory of energy E starting at ϕ_0 and ending at ϕ reads ($\vec{S} = \hbar\vec{s}$, $e = \frac{E}{\hbar^2 m}$):

$$\mathcal{S}_\beta(\phi_0, \phi, e) = \hbar\left[\int_{\phi_0}^{\phi^*}\sqrt{\frac{4e + s^2\cos^2(\phi')}{1+\cos^2(\phi')}}d\phi' + n_\beta\int_0^{2\pi}\sqrt{\frac{4e + s^2\cos^2(\phi')}{1+\cos^2(\phi')}}d\phi'\right] \tag{7}$$

where we have set $\phi^* = \phi_0 + (\phi - \phi_0) \bmod 2\pi$, and n_β is the number of complete revolutions accomplished between ϕ_0 and ϕ ($\phi = 2\pi n_\beta + \phi*$). The

sum runs over classical orbits β of constant energy. This leads us to the expression for the corresponding Green function :

$$\begin{aligned} \mathcal{G}(\phi_0, \phi, e) &= -\frac{i}{\hbar}\sum_\beta \sqrt{|\det \mathcal{D}_{1,\beta}(\phi_0, \phi)|}\exp(\frac{i}{\hbar}\mathcal{S}_\beta(\phi_0, \phi, e) - \frac{i\pi}{2}l_\beta) \\ &=: \sum_n \frac{\psi_n^*(\phi)\psi_n(\phi_0)}{(E - E_n + i\epsilon)} \end{aligned} \tag{8}$$

with

$$\det \mathcal{D}_{1,\beta}(\phi_0, \phi) = \frac{1}{m^2}\left(\frac{\partial^2\mathcal{S}_\beta}{\partial\phi\partial\phi_0}\frac{\partial^2\mathcal{S}_\beta}{\partial e^2} - \frac{\partial^2\mathcal{S}_\beta}{\partial\phi\partial e}\frac{\partial^2\mathcal{S}_\beta}{\partial\phi_0\partial e}\right) = \frac{-4}{m^2\sqrt{(4e + s^2\cos^2(\phi_0))(1 + \cos^2(\phi_0))(4e + s^2\cos^2(\phi))(1 + \cos^2(\phi))}} \tag{9}$$

this result being obtained by partial differentiations of (7). Since this latter value never changes sign, the Maslov index l_β vanishes and so the divergence of the Green function leads to the following semiclassical quantization condition :

$$\int_0^{2\pi}\sqrt{\frac{4e + s^2\cos^2(\phi')}{1 + \cos^2(\phi')}}d\phi' = 2\pi M \tag{10}$$

for any integer $0 \leq M \leq s$. We have then for the averaged density of states (N is the number of states) :

$$\begin{aligned} \overline{\rho}(e) &= -\frac{1}{N\pi}Im\left[\int_0^{2\pi} d\phi\mathcal{G}(\phi, \phi, e)\right] \\ &= \frac{1}{\pi\hbar}\int_0^{2\pi} d\phi\sqrt{|\det \mathcal{D}_{1,\beta}(\phi, \phi)|} \\ &= \frac{2}{\pi\hbar m}\int_0^{2\pi}\frac{d\phi}{\sqrt{(4e + s^2\cos^2(\phi))(1 + \cos^2(\phi))}} \end{aligned} \tag{11}$$

The last equation states in particular that the averaged density of states is proportional to the classical orbit period. Fig. 3 shows the agreement of this semiclassical result with the numerically obtained density of states for the unperturbed quantum model at s=1000. Using (11) we estimate the number of states occupying the regular region of figure 1 :

$$N_{reg} \approx \int_{0.22s^2}^{0.25s^2}\overline{\rho}(e)de \approx \frac{1}{24}(2s + 1) \tag{12}$$

The number of states occupying this region in absence of perturbation is $\frac{1}{24}$ times the total number of states. This gives us a first approximation for the number of regular states we must select. A better approximation in presence of perturbation is given using a method developed by Bohigas et. al. [3]. We must evaluate the number N_{reg} of trajectories that satisfy the condition :

$$\sum_{i=0}^{P} \left[\int_{\phi_i^+}^{\phi_i^-} s_z(\phi') d\phi' + s_z(\phi_i^+)\kappa \right] = 2\pi M \tag{13}$$

for some integers M and P, while it nearly closes on itself after the P^{th} kick, i.e. : $s_z(\phi_M^+) \approx s_z(\phi_0^+)$. ϕ_i^- is the angle between the x and the y component of the spin just before the ith kick while ϕ_i^+ refers to the same angle right after this kick.

This condition means that the action integral must still be an integer multiple of 2 π, and that simultaneously, the orbit must be closed. This condition has meaning only on regular regions were the invariant torii are not destroyed, so that the integrals make sense. We transform this condition and compute the number of trajectories satisfying :

$$\frac{\sum_{i=0}^{P} \left[\int_{\phi_i^+}^{\phi_{i+1}^-} s_z(\phi') d\phi' + s_z(\phi_i^+)\kappa \right]}{\kappa P + \sum_{i=1}^{P} \left(\phi_i^- - \phi_{i-1}^+ \right)} \approx M \tag{14}$$

for integers M and P, and P sufficiently large. With this we replace two conditions by only one numerically more tractable condition. Since our task is to evaluate the number of regular semiclassical levels, and not to determine them precisely, we believe that condition (14) is sufficient. The number of regular states we numerically estimated with (14) is 50 $\pm$ 4 for s=500, i.e. slightly larger than that estimated with (12). In the next section, we will consider this estimated number of regular states as a check of the validity of our selection criterion.

4 Quantum Model

In this section we study the statistical properties of the spectrum of the quantum Hamiltonian (1) for integer spin magnitude. Since the perturbation term is time-dependent, the energy is no longer a good quantum number,

and we are led to define quasi-energies and quasi-energy eigenstates. The Schrödinger equation leads to the following time evolution from right after a kick to right after the next one:

$$\Psi(T^+) = \mathbf{U}_T \Psi(0^+) = \exp(-\frac{i}{\hbar}\kappa \mathbf{S}_z) \exp(-\frac{i}{\hbar}\mathbf{H}^0_{qm} T)\Psi(0^+) \tag{15}$$

Quasi-energies λ and quasi-energy eigenstates Ψ_λ are then defined by :

$$\mathbf{U}_T \Psi_\lambda = \exp(-i\lambda)\Psi_\lambda \tag{16}$$

Since $\mathbf{U}_T$ is unitary, the λ's are real and defined modulo 2 π. We introduce two parities :

$$\mathbf{\Pi}|\mu >= |-\mu > \tag{17}$$

$$\mathbf{\Theta}|\mu >= (-1)^{s-\mu}|\mu > \tag{18}$$

We can express the time-reversal operator $\mathbf{T}$ in term of these two parity operators :

$$\mathbf{\Pi} \circ \mathbf{\Theta}|\mu >= \mathbf{T}|\mu >= (-1)^{s-\mu}|-\mu > \tag{19}$$

In the integer spin case the eigenstates $|\Psi >$ of $\mathbf{U}^0_T := \exp(-\frac{i}{\hbar}\mathbf{H}^0_{qm}T)$ satisfy the conditions :

$$\mathbf{\Pi}|\Psi >= \pm|\Psi > \tag{20}$$

$$\mathbf{\Theta}|\Psi >= \pm|\Psi > \tag{21}$$

So $\mathbf{H}^0_{qm}$ and $\mathbf{U}^0_T$ are in particular time-reversible. The perturbation breaks the $\mathbf{\Pi}$-parity but leaves the $\mathbf{\Theta}$-parity unbroken [5]. We will concentrate on the study of even states, i.e. those states satisfying :

$$\mathbf{\Theta}|\Psi >= |\Psi > \tag{22}$$

However partial results obtained for the odd set of states corroborate the results presented here. The key point is now to find a clear quantum manifestation of the approximate symmetry $S_z \to -S_z$ of the classical phase space

[5]In the half-integer spin case, eigenstates of the unperturbed time-evolution operator are eigenstates of the $\mathbf{\Theta}$-parity only. The latter is left unbroken by the perturbation we consider.

structure. A practical solution is given by Shnirelman's theorem which states that in the semiclassical limit, the quantum states that are confined on the classically chaotic region of the phase space tend to cover it uniformly. To get an insight in this statement we use the following resolution of unity [14]

$$\mathbf{1} = \frac{2s+1}{\pi} \int d\theta d\phi \sin\theta |\theta, \phi >< \theta, \phi| \tag{23}$$

where we introduced coherent states of the spin $SU(2)$ group :

$$|\theta, \phi >:= \sum_{\mu=-s}^{s} \sqrt{\binom{2s}{s-\mu}} \sin(\frac{\theta}{2})^{s-\mu} \cos(\frac{\theta}{2})^{s+\mu} e^{i(s-\mu)\phi} |\mu >$$

These are states that are centered on the point (θ, ϕ) of the sphere and which minimalize the quantum uncertainty. θ is defined by $S_z = S\cos(\theta)$. Using (22), the symmetry of the chaotical region and Shnirelman's theorem [12] :

$$< \Psi_{chaos}|\theta, \phi > \longrightarrow \begin{cases} 0 & \text{on the regular region} \\ const & \text{on the chaotic region} \end{cases} \tag{24}$$

it is then easy to show that

$$< \Psi_{chaos}|\mathbf{s}_z|\Psi_{chaos} > \longrightarrow 0 \tag{25}$$

in the semiclassical limit. This translates into Fig.2 where we plotted an histogram of the expectation value of $\mathbf{s}_z$ taken over quasi-energy eigenfunctions for s=500, $\kappa = 1.1$, z^2=0.5, and m=1. The central peak clearly reflects our reasoning, while the two smaller bumps surrounding it are mainly due to the regular states that are confined to the classical stability islands. The gap in-between is a consequence of the uniform distribution of irregular states. It is remarkable that this gap overlaps the classical frontier between regular and chaotic region.

We used this property to part the irregular states from the regular ones and then study separately the statistical properties of the spectrums of each class of states. We believe this criterion is justified since the fluctuations

$$\Delta\mathbf{s}_z = \sqrt{< \mathbf{s}_z^2 > - < \mathbf{s}_z >^2} \tag{26}$$

of regular states is much smaller than the "Shnirelman gap" appearing in the histogram of Fig. 2 between the huge central peak and the smaller

bumps. As a consequence only very few regular levels will be selected with the set of irregular ones, while maybe more irregular will be counted with the regular ones. Moreover, the fact that the number of selected regular states is in complete agreement with the numerical semiclassical evaluation given by (14) confirms the relevance of this selection criterion.

We now turn our attention to the study of the spectral properties of the time-evolution operator (16). Due to the $\mathbf{\Theta}$-symmetry (18), $\mathbf{U}_T$ belongs to the circular orthogonal ensemble, and not to the circular unitary ensemble as would be expected from the fact that the perturbation breaks the time-reversal symmetry. This situation is similar to the one encountered by Berry & Robnik in certain Aharonov-Bohm billiards [5], or by Delande & Gay in the Hydrogen atom in a magnetic field [10] where the system violates the time-reversal symmetry, but possesses an invariance under a combination of the time-reversal and another symmetry, in our case the $\mathbf{\Pi}$-symmetry. We thus expect a linear repulsion for the part of the spectrum belonging to the irregular states.

The results of our study for a spin magnitude s=500 are plotted in Fig. 4 to 9. Fig. 4 shows a plot of the level spacings statistics for 4233 irregular level spacings computed by diagonalizing ten different evolution matrices for $T = \frac{19}{mS}$ and $1.05 \leq \kappa \leq 1.15$. The solid line is the predicted Wigner distribution. The agreement is excellent. In Fig. 5 we plotted the corresponding cumulative level spacings distribution defined in term of the level spacings distribution [6] $P(\underline{s})$ by

$$W(\underline{s}) = \int_0^{\underline{s}} dt P(t) \tag{27}$$

Also shown are the Poisson and the Wigner distributions. As shown in inset, small deviations from the Wigner distribution appear only around spacings $\underline{s}\approx$2, but have no significance to our opinion.

Fig.6 and 7 show the level spacings and the cumulative level spacings distribution for 572 regular levels taken from twenty different evolution matrices for $T = \frac{19}{mS}$ and $1.05 \leq \kappa \leq 1.15$. The difficulty here is the relatively small number of regular states (≈ 25). Accordingly, only few intermediate or irregular states can have relatively big effects on the statistics. In a convenient

[6] We have used $\underline{s}$ for the level spacings to avoid confusion with the spin magnitude.

basis, $\mathbf{U}_T$ can be represented by the following matrix :

$$\mathbf{U}_T := \begin{pmatrix} R & K \\ K & C \end{pmatrix} \tag{28}$$

$R = R_i\delta_{i,j}$ is a diagonal $\mathrm{N}_{reg}\times$ N_{reg} matrix which corresponds to the N_{reg} regular states, $C = C_i\delta_{i,j}$ is a diagonal $\mathrm{N}_{chaos}\times$ N_{chaos} matrix which corresponds to the N_{chaos} irregular states, and $K = \mathrm{O}(\hbar)$ couples the two subspaces as long as $\hbar$ is finite. In our picture K disappears in the semiclassical limit, and the R_i and C_i satisfy a poissonian statistics and a GOE statistics respectively. It would be of course a hopeless task to try to determine K for finite $\hbar$. The important point is to recognize that as long as $\hbar$ is finite but small enough, K couples only few regular states with irregular ones, this fact resulting in a deviation from the Berry-Robnik surmise. This deviation is then naturally much more important for the regular part of the spectrum, since it contains much fewer levels than the irregular part. We believe that this is the reason for the deviation of the statistics of the set of levels we have selected as regular from the poissonian predicted behaviour. We must recall that our whole reasoning is based on the assumption of two classically homogeneous stability islands. In such a case, semiclassical wave-function would mimic classical orbits and would therefore fit together as concentric circles. The presence of hyperbolic fixed points or cantori may change this picture, possibly turning regular states into intermediate ones as long as $\hbar$ is finite. The semiclassical wave-function overlap and thus interact at certain regions, and this, in the Pechukas picture [1], modify very sensibly the equations of motions governing the evolution of the quasi-energies λ as κ or T is modified, resulting in the appearance of level repulsion. So some intermediate states are phase spatially mixed among the set of states we have selected as regular and consequently modify the corresponding statistics. Their effect is furthermore enhanced by the small ratio of regular levels. A current investigation of the Husimi densities of the selected regular states corroborates this reasoning [15]. Finally we show in Fig.8 and 9 level spacings and cumulative level spacings statistics for the complete set of levels. We compare our results with the Berry-Robnik prediction for a fractional measure of regular states as approximated by (12). The agreement is amazing, and corroborates our picture. The χ^2-test for both graphs - χ^2=25, i.e. half the number of boxes for Fig.8, and χ^2=1480, i.e. 3.3 times less than the number of levels for Fig.9

- gives full statistical significance to these last graphs. We see them as a good evidence for the validity of the Berry-Robnik surmise in our model.

5 Conclusion

We studied the statistical properties of a quantum spin model whose classical counterpart exhibits a mixed phase space configuration. Due to a simple approximate symmetry, whose effect on the quantum system is drastically enhanced by Shnirelman's theorem, we were able to separate the irregular from the regular levels, thereby confirming implicitely the validity of the Percival classification. We then performed a separated statistical study of these levels. The results confirm the Berry-Robnik surmise : while the irregular set of quasienergies exhibits a clear wigner-like shape, the regular part of the spectrum has a clearly different shape, though its spacings distribution does not follow strictly a poissonian law. This deviation is interpreted as the presence of both irregular and intermediate states among the selected regular ones, their effect being enhanced by the relatively small number of the latter. Nevertheless, due to the small number of regular states we believe that the irregular statistics is much more significant, and see our results as a good confirmation of the validity of the Berry-Robnik surmise in our model.

Acknowledgements

One of us (P.J.) gratefully acknowledges fruitfull discussions with J. Bellissard, C. Rouvinez and D. Shepelyansky , as well as the hospitality of the theoretical physics division of the "Laboratoire de Physique Quantique, Université Paul Sabatier" in Toulouse extended to him during his visit when part of this work has been done. We are grateful to T. Prosen for having drawn our attention on reference [7]. Work supported by the Swiss National Science Foundation.

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Figure Captions

Fig.1: Orthogonal projection of the classical phase space on the (S_x,S_y) plane for the case T=$\frac{19}{mS}$, $\kappa = 1.1$ and z^2=0.5.

Fig.2: Histogram of the expectation value of S_z taken over the eigenstates of the unitary time evolution operator defined in (2).

Fig.3: Density of states for the unperturbed Hamiltonian according to (11) (solid line) as compared to numerically obtained datas for the case s=1000 (squares).

Fig.4: Level spacings distribution for 4233 irregular level spacings obtained through direct diagonalization of ten evolution matrices in the parameter range $T = \frac{19}{mS}$ and $1.05 \leq \kappa \leq 1.15$.

Fig.5: Cumulative level spacings distribution for the same case as fig.4. In inset : regions of small deviation relatively to the Wigner-distribution.

Fig.6: Level spacings distribution for a set of 472 regular level spacings obtained through direct diagonalization of twenty evolution matrices in the parameter range $T = \frac{19}{mS}$ and $1.095 \leq \kappa \leq 1.105$. The solid line is the predicted Poisson distribution.

Fig.7: Cumulative level spacings distribution for the same levels as Fig.6 compared to the Poisson distribution.

Fig.8: Level spacings distribution for a set of 5000 regular and irregular level spacings obtained through direct diagonalization of ten evolution matrices in the parameter range $T = \frac{19}{mS}$ and $1.095 \leq \kappa \leq 1.105$. The solid line is the predicted Berry-Robnik distribution with fractional measure of regular states $\rho_1 = 0.08$. χ^2=25 is half the number of boxes.

Fig.9: Cumulative level spacings distribution for the same levels as Fig.8 compared to the poissonian and the Berry-Robnik predicted distribution. In inset : Same curve compared to the Wigner distribution. χ^2=1480 is 3.3 times less than the number of levels.